AMYGDALA

$z \mapsto z^2 + c$

A Newsletter of $\mathcal{M}$ -- the Mandelbrot Set
AMYGDALA, Box 219, San Cristobal, NM 87564
$15.00 for ten issues ($25 overseas)

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THE SLIDE

The slide enclosed with this issue was created by John Dewey Jones, who writes:

"Slide 37 is a 100,000-fold magnification of a miniature Mandelbrot set on the proboscis of the full-size set. I was struck by the progressive increase in the figure's lines of symmetry as one moves from the periphery towards the set in the center — this suggests analogies with many things, for example the increase in the period of oscillations in Verhulst populations until chaos sets in*.

"Image #37 was photographed (1 sec at f8) from the screen of an IBM AT/G. The AT/G screen is 720 pixels wide by 500 pixels high; however, I've used 2 horizontally adjacent pixels for each point in the complex plane in order to extend my palette from the 8 colors available per pixel to a possible 32 per pixel pair. #37 only uses 21 of these 32 shades. In generating the image, I just use the AT/G as a terminal attached to a mainframe. It's directly attached to an IBM 3084, which in turn serves as the front end to a Cray X-MP 24. Given this hardware, I operate in one of two modes: interactively on the IBM, picking out regions to examine with a cursor, or in batch mode on the Cray, using 48-bit precision for each part of each complex number. I chose the Cray becaue single-precision on the IBM gave me a fuzzy image. The calculation took about a minute.

"I use IBM's GDDM graphics software to drive the AT/G, and the APL language to drive the GDDM. Since APL doesn't do sums very fast, I either have it call a Fortran subroutine to do the iterations if we're in interactive mode or pass it the large matrix calculated by the Cray if we're in batch mode."

The picture's coordinates are -1.99641 - 0.000024i (lower left) & -1.99635 + 0.000024i (upper right). The iteration used was $z \mapsto z^2 + c$, with a limit of 1000 iterations.

MORE SLIDES?

JDJ & I have also discussed the "crass economics" of distributing lots of colored slides to subscribers — the problem is that it costs a lot to reproduce slides in quantity 100 or so. He concluded: "Personally, I'd be happy to pay more for the newsletter — $30.00 per year, for example — in order to get more slides. Perhaps you could offer a two-tier subscription rate — $15.00 for the black and white edition, and an additional $15.00 for a color supplement."

I hereby invite subscribers to comment on this proposal. I will regard a $15.00 check as a sincere vote in favor of getting 25 slides (say) over the same period as issues 1-10, as a second-tier subscription for a color slide supplement. I can honor this offer even if only a few subscribers go for it.

Amygdala 1-10 only: US & Canada $15.00; Overseas $25.00.

25 Color Slides only: US & Canada $15.00; Overseas $25.00.

Amygdala 1-10 + 25 color slides: US & Canada $30.00; Overseas $40.00.

WHY “AMYGDALA”?

Several readers have inquired about our use of "Amygdala", what does it mean? I have decided to spill the beans.

“Mandelbrot” is not only the name of the discoverer of $\mathcal{M}$, but is literally "almond bread", in German or Yiddish, naming a particularly delicious slightly sweet cake with almonds in it: mandel = almond + brot = bread. "Amygdala" is Latin for "almond", hence the anatomical term for the almond-shaped brain structure so-called. "Amygdala" is also a perfectly good English word, meaning *almond* (see the Random House Dictionary, e.g.). Thus: Mandelbrot ↦ mandelbrot ⊃ mandel ↦ almond ↦ Amygdala.

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* See e.g. The Beauty of Fractals, pp 23-26. RS

In response to my use of *Amygdala*, George Zopf has coined the adjective *amygdalartoidal*, from αμνγδαλαρτωυσ = almond loaves.

TYPOS

Sigh. In Amygdala #1: Page 1, column 1, line -1 (bottom line), "spring" should be "sprung". Page 3, second figure, the arrow from z+w to z should be reversed. Page 8, bibliographic item #24 is incomplete (See *Bibliography* in this issue).

Those typos were fixed in the second printing, Amygdala #1a, but another typo crept in. The "↦" symbol is available through a special "Mandelbrot" font on the Macintosh, but I printed #1a on a Mac which didn't have that font installed, so instead of "↦", we have "R", e.g. on page 4, column 2, line 5. The "↦" appears OK in the masthead because it's a picture, not text.

TUTORIAL: BASIC MATHEMATICS OF ℳ

In Amygdala #1 we reviewed the definition of the complex numbers as a two dimensional extension of the one dimensional real numbers, and discussed the four basic operations on them: addition, subtraction, multiplication and division.

We mentioned the **modulus** and **argument** of a complex number z = x+iy, but didn't go into detail. The relationship between the real part x, imaginary part y, modulus r, and argument **a**, are as follows:

$x = r \cos a$; $y = r \sin a$;

$r = \surd(x^2+y^2)$; $a = \arctan(y/x)$.

These can be seen as trigonometric relationships in a right triangle — see the second figure on page 2 of Amygdala #1.

The modulus of z is usually notated |z|, and indeed for z a real number, the modulus reduces to its absolute value.

Four properties of the modulus are important for our purposes.

The first is that, given two complex numbers u and v, $|uv| = |u||v|$. A proof involves little more than tedious algebraic manipulation.

The second is the so-called **triangle inequality**: given two complex numbers u and v, $|u+v| \leqslant |u|+|v|$. A proof involves a little more than tedious algebraic manipulation, but is not very profound, and we won't give it here.

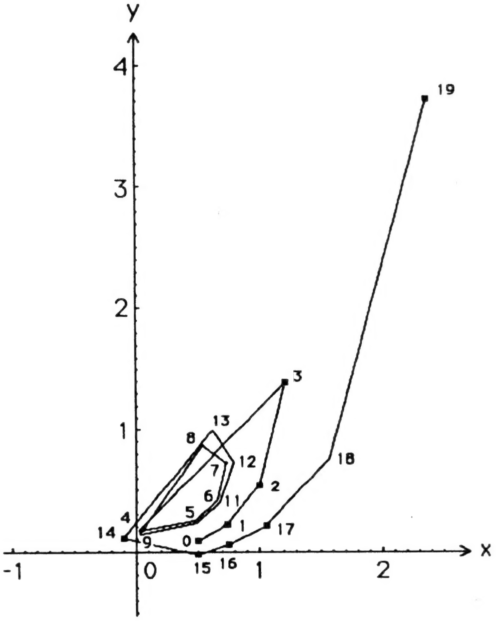
The third is a consequence of the second: $|u-v| \geqslant ||u|-|v||$.

The fourth important property of the modulus is that if $|z| = 2+a$ (where $a > 0$) and $|c| \leqslant 2$, then $|z^2+c| > 2+4a$. This follows from: $|z^2+c| \geqslant ||z|^2-|c|| \geqslant |(2+a)^2-2| = 2+4a+a^2 > 2+4a$.

Next we consider the **quadratic map** $z \mapsto z^2+c$. What does this mean? Given a fixed complex number **c**, the quadratic map $z \mapsto z^2+c$ transforms each complex number **z** into another: $\mathbf{z^2+c}$ by squaring the given z, then adding c. For example, given c = .5+.09i, the quadratic map transforms z = .74+.22i into 1+.54i. It also transforms z = 0 into c = .5+.09i, and z = .5+.09i into .74+.22i.

The basic idea underlying the Mandelbrot set (as well as Julia sets) is applying a quadratic map over and over. Starting with a complex number $z = z_0$, we apply the quadratic map $z \mapsto z^2+c$ to it to produce a second number $z_1 = z_0{}^2+c$. We apply the quadratic map to z_1 to produce $z_2 = z_1{}^2+c$, and so on, producing z_3, z_4, etc. in turn. The successive numbers z_1, z_2, z_3, z_4, etc. are called the **iterates** of z, and the process of applying the quadratic map over and over is called **iteration.**

For example, the figure below shows nineteen successive iterates of the number z = .5+.09i under the map $z \mapsto z^2+c$, where also c = .5+.09i.



The starting point, z_0, is located in the center of the square labeled "0". The successive iterates are labeled appropriately. They cycle around, almost repeating a pattern, before escaping from it and diverging violently. z_{19} is 4.41 units away from the origin, z_{20} is 57.5 units away, z_{21} is 360,000 away, z_{22} is 9.6×10^{16} away, ... The distance from the origin to a number z is just the modulus |z|.

Now the Mandelbrot set. Each number c defines a quadratic map $z \mapsto z^2+c$. Applying that map to z = c itself produces iterates $z_1 = c^2+c$, $z_2 = z_1{}^2+c = (c^2+c)^2+c = c^4+2c^3+c^2+c$, etc. There are two cases: either the successive iterates of c remain bounded in size, or not — there are arbitrarily large iterates. The Mandelbrot set consists of all those c whose successive iterates remain bounded.

As a result of the fourth basic property of the modulus, it is clear that once an iterate of z has modulus > 2, say 2+a, the modulus of the following iterates will exceed $2+4a$, $2+4^2a$, $2+4^3a$, etc., their moduli growing exponentially.

Thus once the modulus of an iterate exceeds 2, we know that the iterates are unbounded. Consequently we have the famous test for membership in $\mathcal{M}$: z is in $\mathcal{M}$ if and only if all its iterates z_n have $|z_n| \leqslant 2$.

For the c of the diagram (c = .5+.09i), the nineteenth iterate has modulus 4.41, hence c is not in $\mathcal{M}$.

(to be continued)

FRACTAL SOFTWARE REVIEWS

Most Amygdala subscribers have computers and are interested in programs which generate fractal images. At this point I have been sent eight diskettes containing fractal software. I would like to print reviews of their contents, but alas! I can only handle the Macintosh ones, so I'd like to propose the following:

(1) For each computer X used by a nonempty set of subscribers, let one subscriber volunteer to be the contact ("Contact X") for fractal software for computer X. I will print the names and addresses of these contacts.

(2) If you create or acquire a fractal software diskette (FSD) for computer X, and want to share it with other Amygdala readers, you should send it to Contact X.

(3) Contact X will examine and digest each FSD sent to him, and send a short review for publication in Amygdala. The review should include information on where copies of the diskette may be obtained.

(4) Contact X agrees to provide copies of FSDs which are freeware or shareware to subscribers on request for a nominal fee (I suggest $5.00 - $10.00).

(5) If the workload for computer X grows too heavy, I will supply to Contact X the names of alternates who are willing to share the load.

At this point, I have the following:

For the PC:

Mandelbrew (Platt)

Superman (Marshall Dudley)

For the Sperry HiRes Graphics System

Mandelbrot Set Generator disk & Picture Disk #1

For the Macintosh:

Four diskettes, to be reviewed in a later issue.

AN ALGORITHM FOR APPROXIMATING THE MAIN REGION OF THE MANDELBROT SET

Frank A. Browne

The "main" region of the Mandelbrot set resembles a somewhat flattened heart, rotated 90 degrees. The point c = -.5+.25i , for example, is interior to this region, but the points -1+0i and -.125+.75i are not.

Iterations within the main region are characterized by rapid, monotonic convergence of the magnitude of z to a final value, which I call $|z_\infty|$. The real and imaginary parts of z perform exponentially decaying oscillations around their respective steady-state values x_∞ and y_∞. A simple procedure for approximating the outline of the main region — without any of the "nubs" or appendages that are so interesting — is to find the locus of points for which these oscillations neither grow nor decay. The use of a *differential* (small-oscillation) criterion for growth/decay makes the problem readily manageable. The result is shown at the bottom of the next page — the black line superimposed on a gray picture of a portion of $\mathcal{M}$. The paragraphs that follow provide a derivation of the algorithm.

Problem Analysis — The steady-state quantities x_∞ and y_∞ must satisfy the equations

$$x_\infty = x_\infty{}^2 - y_\infty{}^2 + x_0 \qquad (1)$$

$$y_\infty = 2\,x_\infty y_\infty + y_0 \qquad (2)$$

where $x_\infty + y_\infty i = z_\infty$, and $x_0 + y_0 i = c$. The solutions are:

$$x_\infty = (1 - |y_0|/y_\infty)/2 \qquad (3)$$

$$y_\infty{}^2 = 1/8(-(1-4x_0) + \surd[(1-4x_0)^2 + 16y_0{}^2]) \qquad (1)$$

The issue of the correct sign to use when taking the square root of $y_\infty{}^2$ does not have to be resolved for the purposes of this analysis.

The complex number version of Eqs (1) and (2),

$$z_\infty = z_\infty{}^2 + c$$

implies that you will get the response

$$\Delta z_\infty = 2\,z_\infty\,\delta z_\infty$$

to a small perturbation δz_∞ in the steady-state value z_∞. This result is useless as it stands, but it suggests another approach which shows that

$$|\Delta z_\infty| = 2\,|z_\infty|\,|\delta z_\infty| \qquad (5)$$

Let's now make the obviously false assumption that the boundary between growing and decaying iterations is characterized by *small* steady-state oscillations about x_∞ and y_∞. This would require that $|\Delta z_\infty| = |\delta z_\infty|$, which in view of Eq(5) requires that $|z_\infty| = 1/2$, or

$$|z_\infty|^2 = 1/4 \qquad (6)$$

The locus of points (x_0, y_0) for which this is true is our simple approximation to the boundary of the Mandelbrot set. Fortunately, a formula for y_0 in terms of x_0 can be obtained in closed form, as follows.

Formula Derivation — Rearrange the terms of Eq(1) to obtain

$$y_\infty{}^2 = x_\infty{}^2 - x_\infty + x_0 \qquad (7)$$

Eq (6) implies that $x_\infty{}^2 + y_\infty{}^2 = 1/4$, or $y_\infty{}^2 = 1/4 - x_\infty{}^2$. Substituting this last result into Eq (7) and collecting terms, we get $2x_\infty{}^2 - x_\infty - (1/4 - x_0) = 0$, whose solution is

$$x_\infty = 1/4\,[1 \mp \surd(1 + 2(1 - 4x_0))] \qquad (8)$$

You can see that the largest possible value for x0 is $x_0$max = 3/8.

The choice of sign for the radical in Eq (8) is to be made as follows:

For $1/4 \leqslant x_0 \leqslant 3/8$ use the MINUS sign (near the cusp).

For $3/8 \geqslant x_0 \geqslant -3/4$ use the PLUS sign (rest of the curve).

From Eq(2) we can derive

$$y_\infty = y_0/(1-2x_\infty) \qquad (9)$$

Now take the following steps: square both members of Eq (9); substitute Eq (8) into the result; equate the right-hand member of the last result to the right-hand member of Eq(4). The result to this point is

$$\frac{V}{1 + U \mp \sqrt{1 + 2U}} = -U + \sqrt{U^2 + V} \qquad (10)$$

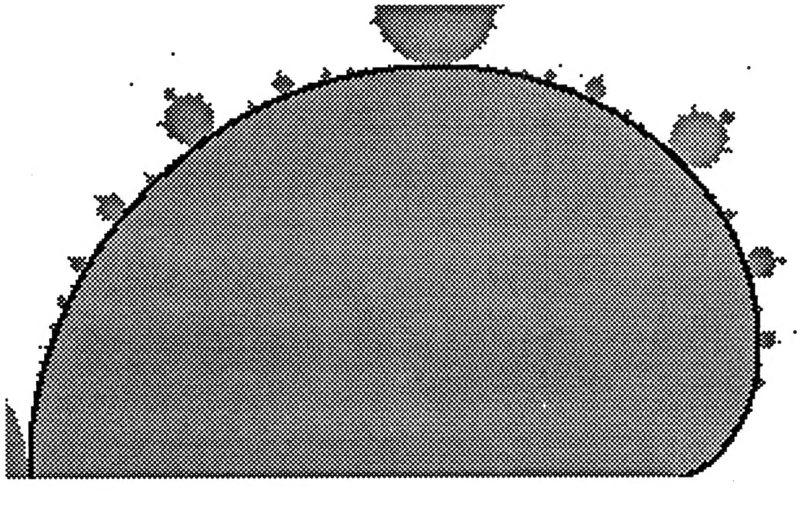
where $U = 1 - 4x_0$ and $V = 16\,y_0{}^2$.

Now transposing the -U term to the left-hand side and squaring both sides, terms in U drop out. After dividing both sides by V and doing some algebra, we get the desired result:

$$V = -U^2 + 2U + 2 \mp 2\surd(1 + 2U) \qquad (11)$$

In terms of x_0 and y_0, the final product is

$$16y_0{}^2 = 3 - 16x_0{}^2 \mp 2\surd[1 + 2\,(1 - 4x_0)) \qquad (12)$$



Here the sign of the radical is to be chosen as noted above. Eq (12) was programmed to produce the curve shown in the figure below.

The curve is shown drawn in black, superimposed on a portion of $\mathcal{M}$ drawn in gray.

Limiting points on the curve can be shown to be as follows:

$1/4 + 0i$	(the cusp)
$3/8 + \surd 3i/8$	(farthest to the right)
$-1/8 + 3\surd 3i/8$	(top & bottom)
$-3/4 + 0i$	(farthest to the left)

Very near the cusp the equation can be approximated by $16y = -U^3$. From this you can show that the arcs emanating from the cusp make an angle of zero at the cusp, despite the visually apparent angle of 60 degrees or so that they soon attain.

- -

I [RS] suspect that the shape of the cusped curve can be analyzed by studying the ralationship between the maps $z \mapsto z^2-\mu$ and $z \mapsto \lambda z(1-z)$. The cardioid-like region associated with $z \mapsto z^2-\mu$ corresponds to a circular region for $z \mapsto \lambda z(1-z)$, and the formal transformation from λ-map to μ-map should transform the circle into a cardioid, or whatever it is. See "The Fractal Geometry of Nature", pp 188, 189, 186.

COMPUTERS, PATTERN, CHAOS & BEAUTY

Clifford Pickover

The author sent me a copy of this remarkable paper, published as an internal memorandum at the IBM Thomas J. Watson Research Center. Here is the abstract:

> To help characterize complicated physical and mathematical structures and phenomena, computers with graphics can be used to produce visual representations with a spectrum of perspectives. This paper is a "road map" of the author's graphics perspectives developed in the last few years. The applications are varied and include the diverse fields of speech synthesis, molecular biology, mathematics and art. Often the shapes representing physical phenomena are of interest both artistically and scientifically, and they reveal a visually striking and intricate class of patterns. The use of "lateral thinking" when exploring computer applications is also described.

This is one of the most interesting papers I [RS] have read in a long time, and I recommend it unhesitatingly. Its range is much broader than the subjects of fractals and the Mandelbrot set, but these areas appear as well as others. Of particular interest are the pictures of iteration maps of trancendental functions, including a remarkable "Biomorph zoo", which contains "microorganisms" like one might see under a microscope — except for the fact that most of the organisms are made up of organelles which are tiny replicas of themselves!

Copies may be requested from:

IBM Thomas J. Watson Research Center
Distribution Services 73-F11
PO Box 218
Yorktown Heights, NY 10598

A LETTER FROM B.B. MANDELBROT

I sent our hero, Benoit B. Mandelbrot, a copy of Amygdala #0, and received this letter in reply:

"Dear Mr. Silver,

"Thank you very much for informing me of the existence of AMYGDALA and for sending issue #0. I had tried in the past to motivate some people to take this initiative, but I did not really try very hard, and nothing happened. I am pleased that your initiative comes from you alone.

"May I inquire about your background and other activities? I propose to put your initiative in the Fractals mail within IBM to inform my younger colleagues. I shall also look around for material for your BIBLIOGRAPHY column. I enclose my own papers on the topic, together with an expository paper and an interview that my be relevant.

"The above address is good til January 1. After that, I shall return to Harvard, where I teach in the spring.

"Having taken many years of Latin as a boy, I did not need to go to the dictionary to understand your title. But how does an American boy think of Latin?

"Sincerely.

"Benoit B. Mandelbrot"

I've taken this letter as a point of departure for three features in this issue: the provenance of the name, "Amygdala", the four bibliographic references to the papers which BBM enclosed, and my review of one of these papers.

REVIEWS

BB Mandelbrot. "**On the Quadratic Mapping $z \to z^2-\mu$ for Complex μ and z: the Fractal Structure of its $\mathcal{M}$ Set, and Scaling**". Physica 7D (1983) 224-239 [Cited as #28 in the **Bibliography**, this issue.]

The content of this highly informative paper are perhaps best summarized by simply citing its abstract and introduction, as well as the headings of its three sections.

Abstract

For each complex μ, denote by $\mathcal{F}(\mu)$ the largest bounded set in the complex plane that is invariant under the action of the mapping $z \to z^2-\mu$. Mandelbrot 1980, 1982 (Chap. 19) reported various remarkable properties of the $\mathcal{M}$ set (the set of those values of the complex number μ for which $\mathcal{F}(\mu)$ contains domains) and of the closure $\mathcal{M}^*$ of $\mathcal{M}$. The goals of the present work are as follows. **A)** To restate some previously reported properties of $\mathcal{F}(\mu)$, $\mathcal{M}^*$ and $\mathcal{M}$ in new ways, and to report new observations. **B)** To deduce some known properties of the mapping $\mathcal{F}$ for real mu and z, with $\mu \in [-1/4,2]$ and $z \in [-1/2-1/2\sqrt{(1+4\mu)},1/2+1/2\sqrt{(1+4\mu)}]$. In many ways, the properties of the transformation $\mathcal{F}$ are easier to grasp in the complex plane than in an interval. (This exemplifies the saying that "when one wishes to simplify a theory, one should complexify the variables".) **C)** to serve as introduction to some recent pure mathematical work triggered by Mandelbrot 1980. Further pure mathematical work is strongly urged.

Introduction

The [17] illustrations are the focus of this paper, and the text is organized around the illustrations, in the form of extended comments.

1. Discussion of figures 1a to 1e. Illustration of the action of $z \to f(z,\mu) = z^2-\mu$ on a large complex circle. Sequences of algebraic curves approximating the repeller (Julia) sets $\mathcal{F}^*(\mu)$.

2. Discussions of figures 2a to 2f. Classification of the values of μ by the topology of $\mathcal{F}(\mu)$. The Sets $\mathcal{M}$ and $\mathcal{M}^*$. Sequences of algebraic curves approximating $\mathcal{M}^*$. The continent, islands, stellate structures, devil's causeways.

3. Discussion of figures 3a to 3f. The repeller stack for real μ and complex z. Illustrations of the influence of the value of μ on the shape and the topology of the repeller (Julia) set $\mathcal{F}^*(\mu)$.

The references in the abstract are to (Mandelbrot 1982) **The Fractal Geometry of Nature**, and (Mandelbrot 1980) the paper cited as #27 in the **Bibliography**, this issue.

Timothy Hammett of New Zealand sent the following review and comment.

Dick Pountain. "**Turbocharging Mandelbrot**". Byte 11 (September 1986) 359-366. [Item #20, BIBLIOGRAPHY]

The author describes the (software) techniques required for programming the Mandelbrot calculation on an array of Inmos transputers (which are parallel processors). The time he quotes for an average Mandelbrot on 11 transputers (9 doing the calculations, 1 for control and 1 for graphics output) are 20-60 seconds for a 256 x 256 image (VAX ~1hr, IBM PC, NO 8087 ~ 4-8 hrs) — I saw a small item about it on TV and can verify the speed.

LETTERS

From Daniel R. Buskirk
The Rockefeller University / 1230 York Ave
New York, NY 10021-6399

I would like to take this opportunity to make a program disk available to your readers. The main program gives the user a chance to observe the iterates of a single M-set point against a silhouette of the set. The many symmetries of the iterates are fascinating and mathematically significant. This program requires an IBM PC or compatible with CGA graphics. The program disk also includes C source code for set generation and some various Mandelbrot goodies. I will send a disk postpaid in the US for $10.

BIBLIOGRAPHY

Items #24-25 were submitted by Allen B Freedman, #26 by Thomas Bank. #27-30 cite four papers, reprints of which were kindly sent in by BB Mandelbrot, as was #34.

24. M van de Panne. "3-D Fractals". Creative Computing 11 (July 1985) 78-82. [Includes BASIC programs for TRS-80 Models I/II/III/4 (with CGP-115 plotter), Macintosh, and Apple II]

25. M Zeidenberg. "Snowflakes and Dragons: Drawing Fractals with Macintosh Pascal". Macworld 2 (August 1985) 124-131. [Includes MacPascal programs "Koch", "Dragon", and "Snowflake"]

26. HE Stanley & N Ostrowsky (eds). On Growth and Form: Fractal and Non-Fractal Patterns in Physics. Martinus Nijhoff Publishers, 1986. [This is a rather technical report on a conference held in Corsica, France, from 26 June through 6 July 1985. Although it does not discuss the Mandelbrot Set in particular, it has in depth discussions and research results on fractals in general.]

27. BB Mandelbrot. "Fractal Aspects of the Iteration of $z \to \lambda z(1-z)$ for Complex λ and z". Annals of the New York Academy of Sciences 357 (26 December 1980) 249-259.

28. BB Mandelbrot. "On the Quadratic Mapping $z \to z^2-\mu$ for Complex μ and z: the Fractal Structure of its $\mathcal{M}$ Set, and Scaling". Physica 7D (19830 224-239. [See review on page 5]

29. BB Mandelbrot. "On the Dynamics of Iterated Maps III: The Individual Molecules of the $\mathcal{M}$-set, Self-Similarity Properties, the Empirical n^2 Rule, and the n^2 Conjecture". In Chaos, Fractals, and Dynamics, P Fischer & WR Smith (eds), Marcel Dekker, Inc. (1985).

30. BB Mandelbrot. "On the Dynamics of Iterated Maps VIII: The Map $z \to \lambda(z+1/z)$, from Linear to Planar Chaos, and the Measurement of Chaos". In Chaos and Statistical Methods, Y Kuromoto (ed), Proceedings of the Sixth Kyoto Summer Institute, Kyoto, Japan, September 12-15 1983; Springer-Verlag (1984).

31. MM Novak & J Weber. "Fractal Sets". PCW (PC World?) (December 1986) 196-199.

32. H. Katz. "A Mandelbrot Program for the Macintosh". Dr. Dobb's Journal (November 1986) 42-45, 71-87.

33. I Peterson. "Parallel Paths". Science News (January 10, 1987) Front cover, 28-30) [Discusses "massively parallel" computers. Cover: Mandelbrot view of a 30-legged Ant]

34. A Barcellos. "Benoit Mandelbrot". Mathematical People, Profiles and Interviews, DJ Albers & GL Alexanderson, eds. Birkhauser (publisher). [A 19 page interview in which BB Mandelbrot talks about his life and work.]

35. Clifford A Pickover. "Computers, Pattern, Chaos and Beauty". IBM Thomas J. Watson Research Center; Yorktown Heights, NY 10598. [See review on page 4]

36. H-O Peitgen, D Saupe, F v Haeseler. "Cayley's Problem and Julia Sets". The Mathematical Intelligencer (6,4) 1984. p11 - 20.

MAIL-ORDER BOOKS ABOUT FRACTALS: **THE BEAUTY OF FRACTALS** lists for $35.00, and **THE FRACTAL GEOMETRY OF NATURE** lists for $34.95. Amygdala subscribers can get either one from us for $31.50, postpaid. Please allow 4-6 weeks for delivery, ARO.

COMMERCIAL PRODUCTS

ART MATRIX; PO Box 880; Ithaca, NY 14851 USA. (607) 277-0959. Prints, FORTRAN program listings, 36 postcards $7.00, sets of 2 packs $10.00, 140 slides $20.00. Or send for FREE information pack with sample postcard. Custom programming and photography by request. Make a bid.

Recreational & Educational Computing Newsletter; Dr. Michael W. Ecker, Editor; 129 Carol Drive; Clarks Summit, PA 18411.

RECN is devoted to the playful interplay of computers and math -- "mathemagic". Martin Gardner-like in style, but adapted to computers, REC has featured articles on a variety of subjects, plus the art of gifted individuals such as mathematician-artist Dr. Nathaniel Hellerstein; editorials, humor, programming chalenges and tips, reader solutions, and more.

Subscription: $18.95 for 1987; $35 for 1986+1987.

A reader of Amygdala may have a sample issue for $2 if he so identifies himself.

CIRCULATION

As of today (March 26, 1987) Amygdala has 114 paid-up subscribers and the newsletter will be sent gratis to 10 others — until they pay up!